Kinematic Mount for Deployable Tertiary

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1 Introduction

We want to support the tertiary mirror in a fashion that is stable and very repeatable. Specifically we want the tip- tilt to repeat to ~10 arcsec (0.87 arcsec in the focal plane) (48μm across 1m) and piston to ~0.1mm. Motion in the plane of the tertiary mirror does not matter since the mirror is flat.

We also want the alignment of the tertiary to not change much as the telescope moves from zenith to horizon. The telescope itself is not perfectly stiff; corrections of order 15 arcsec are experienced, so zenith angle dependent changes in the pointing at the level of ~5 arcsec are acceptable for the tertiary. Of course repeatability at a much smaller level is needed.

Pupil stability at the level of 0.001 of the diameter requires that the tertiary mirror not change in tilt more than about 11 arcsec, either as a function of zenith angle or repeatability.

2 Three strut option

This can be achieved with 3 struts that are normal to the plane of the tertiary. Perhaps the key issue here is what these 3 struts are attached to. When the mirror is retracted these struts must not block the cassegrain field of view and need to stiffly attach to the bearing system when the mirror is deployed. Since the line of force is along the struts and normal to the deployed mirror, these forces are not in the cylinder defined by the bearings. The 3 struts might work in bending to provide adequate stiffness for the 3 in-plane motions.

3 Three v-grooves

Another approach that seems more practical is to use a full kinematic mount constraining all 6 degrees of freedom of the tertiary. Here we can imagine 6 struts that provide the needed stability. Almost any pattern of 6 struts works in principle, and once attached will provide stability. Achieving repeatability is more difficult. One promising approach is to use 3 v-grooves with balls to provide the attachment. The concept is that of 3 grooves in a rigid body and one places the object of interest (with 3 balls attached) on this rigid body, the 3 balls going into the 3 grooves, each ball contacting its v-groove at 2 places, thus providing 6 repeatable constraints.

In our situation, the simplest geometry is to attach each v-groove to the bearing system with 2 struts. A convenient geometry will have the two struts forming a plane. Since conceptually the v-grooves only provide 2 constraints it is critical that if 2 struts support a v-groove, that the plane of the struts is normal to the v-groove axis. Otherwise small motions along the v-groove will cause the mirror to tilt. This will have an adverse impact on repeatability. This also applies to small motions of the v groove itself that will be caused by out-of-plane bending of the struts. Only when the v groove axis is normal to the strut plane will this motion be unimportant.

In general this means that the 3 v-groove axes will not lie in a plane. It also means that the plane defined by the two struts may not be parallel to the bearing axis. This may not matter so long as adequate stiffness in the 3 key degrees of freedom is achieved. A virtue of this approach is that the 2 struts supporting a v-groove block need not be in a straight line as projected along the cylinder, thus there can be less blockage.

Rather than work out the equations, it is probably simpler to use FEA to assess performance.

4 Simple Examples

4.1 Single axially loaded strut

Consider a strut of steel

E=2e11Pa

L=0.7m

A=1e-4 m^2

If we assume we are supporting 100kg with 3 struts, F=100*9.8/3=327N

So the strut compresses under the specified force by

$$\delta = \frac{FL}{EA} = 11\,\mu\text{m}$$

The mass of a single strut is only 0.55 kg

Of course it is easy to increase the cross sectional area if we want smaller deflections.

4.2 Thermal Impact

Let us change the temperature (or rather a temperature difference between struts) by 1deg.

Let α=1.3e-5

We get a length change of

$$\delta = \alpha \Delta T L = 12\,\mu\text{m}$$

This may seem excessive; if so we might use Invar to reduce the thermal effects.

4.3 Deflections of two struts

Consider two struts that support a v-groove block. Assume they are both the same length and come together at the v-groove block. Assume both are connected to a base (one of the bearings) and their separation at the base is 2b. Let the height of the v-block above the base be h. Then the strut length L is

$$L = \sqrt{b^2 + h^2}$$

It is straightforward to find the stiffness of this bipod in the plane of the bipod both parallel to the base and perpendicular to the base as

$$\delta_{perpendicular} = \frac{FL}{2EA}\frac{L^2}{h^2}$$

$$\delta_{parallel} = \frac{FL}{2EA}\frac{L^2}{b^2}$$

thus if b=0.3m and h=0.7m L=0.762m and for a force of F=327N, A=1e-4m^2 we find

$\delta_{perpendicular}$=7.38µm

$\delta_{parallel}$=40.2µm

Clearly we want as large a base as practical or we want the parallel motion to not have much impact on the tilt of the tertiary.

Now consider the out-of-plane motions (bending). The deflection for a given force F at the end is given by

$$\delta_{bending} = \frac{FL^3}{3EI}$$

If we use a cylindrical tube of outer radius r_o the inner radius is given by

$$r_i = \sqrt{r_o^2 - A/\pi}$$

and for a single strut

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(r_o^2 - r_i^2)(r_o^2 + r_i^2) = \frac{A}{4}(r_o^2 + r_i^2)$$

so for the bipod (two struts) we get

$$\delta_{bending} = \frac{FL^3}{6EI} = \frac{FL}{2EA}\frac{4L^2}{3(r_o^2 + r_i^2)}$$

now assume the force (along the axis of the v groove)is limited by friction to $F_{tan} \leq \mu F_{normal}$. For a normal force=100*9.8/3=327N and μ=0.2 we gat F=65.3N. If we pick A=1e-4m^2 and r_o=12.5mm, we get r_i=11.15mm and $\delta_{bending}$=3.34e-3m, probably a bit too large.

If we increase the area to A=4e-4m^2 and r_o=0.025m we get r_i=0.0223m and $\delta_{bending}$=2.14e-4m a more acceptable value.

The v groove should in principle be orthogonal to the plane formed by the supporting struts. If so, then any bending simply moves the v groove along its axis, so no change occurs. If the v groove is tilted then the deformations from bending will cause motion of the v groove, and tilt of the tertiary mirror. An angular error of dθ will cause motion of the mount point by $d\theta\delta_{bending}$ which for our example above means that an angular error of 0.01 radian produces 2μm of vertical motion, or a tilt of the tertiary of ~2e-6m/0.5m~0.8 arcsec, probably acceptable.

It is also of some interest to know how much the cantilever beam moves in the direction of the plane. It can be shown that for a point load at the end of the beam, the radius of curvature that defines the end motion is R=5/6*L where L is the beam length. Thus the motion in the plane is

$\delta L = \frac{3\delta_{bending}^2}{5L}$ which for the deflection $\delta_{bending}$=2.145e-4m, gives δL=0.036μm which is very small. Since this is quadratic in the bending deflection, if we can reduce the coefficient of friction by a little, this component of mirror motion will be even more negligible.

Some simplicity can be achieved if we arrange the struts so they are tangent to the cylinder. This should provide the maximum stiffness since the forces will be applied to the cylinder directly in the cylindrical surface. Doing this will place the node (the kinematic mount) outside of the cylinder by

$$x = R_c \frac{1-\cos\theta}{\cos\theta}$$

where R_c is the radius of the cylinder (~0.6m)

and θ is the half angle the strut makes to the line of symmetry ($\tan\theta = b/R_c$= 0.3/0.6, θ=26.6 deg)

so x=0.071m and the v groove is inclined by tanϕ=x/h=0.071/0.7, ϕ=5.78 deg.

4.4 Effects of friction on 3 v grooves

Now we consider 3 balls mounted in 3 v grooves. For simplicity assume the v's are 90 deg, symmetrically oriented, the grooves point to a common point and the grooves are 120 deg apart. Further, assume the mirror is centered on 3 balls that form an equilateral triangle and that gravity is acting normal to the mirror and the plane of the balls.. In the absence of friction the system will move until the balls are in the grooves and making contact with both faces of the v's at which point the system will be held rigidly in a repeatable location.

However, if friction is high enough, the system may stop short of this minimum energy state. We will use conservation of energy to find the critical coefficient of friction.

The worst case will have 2 balls in their v grooves (lowest energy state), and the 3rd ball has contacted one face of the third groove but has not yet slid down to contact the other face. If the system is close to its final state the only motion that is allowed by the two balls in their grooves is rotation about a point P. That point is the intersection of the two lines that pass through the final ball positions and are perpendicular to the two v grooves.

Assume the two balls move a small distance u to put them in their desired location. The work done is then

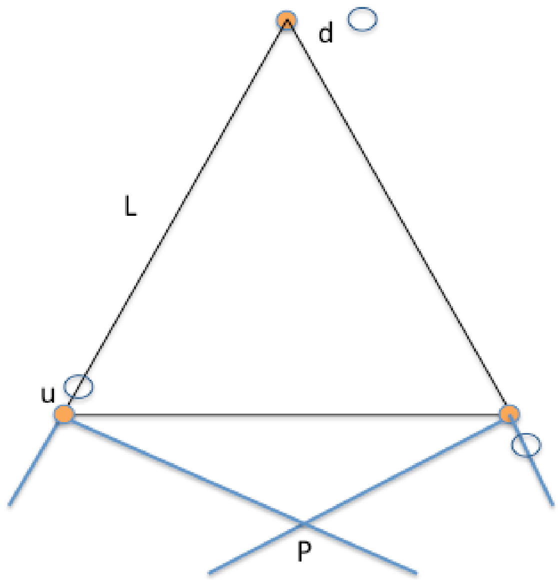
$$W_{1+2} = 4\mu F_{normal} u$$

where

$$F_{normal} = \frac{Mg}{3\sqrt{2}}$$

and the third ball must move a horizontal distance

$$d = \left(\frac{\sqrt{3}}{2}L + \frac{L}{2}\right)\frac{\sqrt{2}u}{L} = \frac{1+\sqrt{3}}{\sqrt{2}}u$$



The work it does sliding down the inclined surface is thus

$$W_3 = (\sqrt{2}d)\mu F_{normal}$$

and the potential energy is

$$E = mgh = \frac{Mg}{3}d = \frac{Mg}{3}\frac{1+\sqrt{3}}{\sqrt{2}}u$$

At the critical coefficient of friction all the potential energy is spent in fictional heating and the 3rd ball hits the side of the v groove at zero velocity, thus

$$W_{1+2} + W_3 = E$$

$$4u\mu \frac{Mg}{3\sqrt{2}} + \sqrt{2}\mu \frac{Mg}{3\sqrt{2}} \frac{1+\sqrt{3}}{\sqrt{2}} u = \frac{Mg}{3} \frac{1+\sqrt{3}}{\sqrt{2}} u$$

or

$$\mu = \frac{1+\sqrt{3}}{5+\sqrt{3}} = 0.406$$

if the coefficient of friction is greater than this the kinematic system will not work, less than this, the system will slide into proper position under its own weight.

4.5 Orientation of the v groove

Nominally we want to orient the v groove facing upwards in a symmetrical fashion; each face is 45 deg from the vertical. If this is not exactly true what happens? The frictional force needed to slide the ball along the v groove is given by

$$F_\mu = \mu Mg(\cos\theta + \sin\theta)$$

where θ is the angle a face of the v groove makes to the horizontal. At the nominal design θ=45 deg, and the function is at its maximum, decreasing symmetrically as the v groove rotates until one face is horizontal.

We have been assuming that the three v grooves lie in a plane, but with a bit more generality we can assume they lie on the surface of a cone, all pointing to the apex. In fact, except for singular cases, the v grooves can have any orientation at all, and they will still act as kinematic mounts. The most general case will impose some constraints on the allowed coefficients of friction, but the formulas are not readily available.

4.6 Bending of support of the mirror assembly

No matter how the mirror assembly is ultimately connected to the bearings, the support system must carry the weight across the span of the bearings. For a simply supported beam loaded at the center the deflection is

$$\delta = \frac{FL^3}{48EI}$$

We work out a representative example with

L=1.4m

ρ= 7.8e3 kg/m^3

F=100*9.8/3=327 N

We will use 3 box beams to carry the load, assume that each has a mass of 5 kg and a depth of h=0.1m. Although deeper is better, we must remember that it should not block the beam when retracted. For a box beam whose width is equal to its depth, A=4wh and M=ρAL/2=2ρwhL. So w=2.29mm and A=9.16e-4m^2 and

$$I = \frac{h^4 - (h-2w)^4}{12} \cong \frac{2}{3} w h^3 = \frac{2}{3} \frac{A}{4} h^2$$

So we get I=1.53e-6m^4

And δ=61.1μm.

This is innocuous if it is only piston with no significant rotation.

If the beam experiences a moment at the center, such as might happen if the mirror and its support system has its center of mass displaced from the center line of the supporting beam then the mirror will also rotate by

$$\theta = \frac{ML}{24EI}$$

where M is the moment applied at the center of the beam

If we want the rotation of the tertiary to be no more than 10 arcsec at the horizon, then we need the cg offset to be no more than

$$u = \frac{24EI}{mgL} \theta$$

or for m=100kg, L=1.4m,

u=0.26m

a value that should be easy to achieve.