K1DM3 Design Note
Tertiary Kinematic Mounts

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1 Kinematic mounts

We consider mounts that resemble a v-groove and a ball or cylinder let the half opening angle of the v-groove be θ and the applied force at an angle ϕ relative to the plane of symmetry of the v-groove. The ball will slip if cot(θ+ϕ)>μ. For dry steel μ may be as large as 0.7 but with suitable lubricants we can keep μ<0.2.

1.1 Contact stresses

d= depth of indentation

a=contact radius

$$a = \sqrt{Rd}$$

$$F = \frac{4}{3} E^* R^{1/2} d^{3/2}$$

$$\frac{1}{E^*} = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}$$

1.2 Contact between elllipsoids

$$\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}}$$

$$\frac{1}{R_y} = \frac{1}{R_{1y}} + \frac{1}{R_{2y}}$$

$$\frac{1}{R} = \frac{1}{R_x} + \frac{1}{R_y}$$

contact ellipse has dimensions

$$a = \sqrt[3]{\frac{3k^2 EPR}{\pi E^*}}$$

$$b = \sqrt[3]{\frac{3EPR}{\pi k E^*}}$$

$$p_0 = \frac{3P}{2\pi ab}$$

where P is the applied force, and

$$k = 1.0339[\frac{R_y}{R_x}]^{0.6360}$$

$$E = 1.0003 + \frac{0.5968 R_x}{R_y}$$

these are approximate, the exact formulas involve elliptic integrals. It is convenient to use http://www.mesys.ch/calc/hertz.fcgi for actual calculations.

1.3 Stresses between sphere and plane

$$p(r) = p_0 (1 - \frac{r^2}{a^2})^{1/2}$$

$$p_0 = \frac{3F}{2\pi a^2} = \frac{1}{\pi}(\frac{6FE^{*2}}{R^2})^{1/3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

now for identical materials (steel) and a half plane

E=2.0e11 Pa

ν= 0.3

so E^*=1.1e11Pa

Yield strength =700e6 Pa for high strength steel, up to 2600e6Pa for Maraging steel. Carpenter Speed Star steel has a yield strength of roughly 3300MPa.

1.4 between two parallel cylinders

$$F = \frac{\pi}{4} E^* L d$$

$$a = \sqrt{Rd}$$

$$p_0 = (\frac{E^* F}{\pi L R})^{1/2}$$

2 Tertiary mirror

The existing tertiary is on a bearing with an id of 1180mm and an od of 1218mm. Chris says that the tower will allow at least 1270mm diameter.

Sensitivity to slope errors is not the same as with near normal incident mirrors. Of course angle of incidence equals angle of reflection, so the motion of the mirror normal is important. There are two coordinate systems of primary interest: the tertiary itself and the telescope elevation structure.

Rotation about the telescope z axis moves the image by the angle*lever arm (the distance from the tertiary to the focus, here 6.5m). Rotation of the tertiary about the y axis moves the image by 2*angle*lever arm. Rotation of the tertiary about the x axis moves the image by =angle*lever arm.

Similarly in the tertiary mirror coordinate system (z is normal to the surface, u is along the ellipse major axis) rotation about z does nothing, rotation about v causes image motion =2*angle*lever arm. Rotation about u causes image motion =2*cosθ* angle*lever arm. Where θ is the tilt of the tertiary (45deg)

We want the mirror and its support system to introduce rms errors on the sky no more than 7.3e-8 radians.

In the plane of the tertiary this means that rms slope errors along u must be under 8.41e-7 radians and in the v direction rms slopes errors must be under 1.19e-6 radians.

0.881x0.623m

$A=0.431m^2$

h=0.050m

M=54.5 kg

If we assume the support structure is significant, then the force needed to support the tertiary against gravity is ~1000N

3 Examples

3.1 Sphere on Plane

Consider a sphere on a plane R=0.0125m, E^*=2.2e11 Pa, L=0.025m

Let the critical stress be p_0=3.5e8 Pa, beyond which damage to the material may occur. So

$$F_c = \frac{\pi^3 p_0^3 R^2}{6E^{*2}}$$

=0.72N

put another way if R=0.0 25m and we assume the mirror plus support is 100kg and assume a load on the kinematic mounts that's 5x gravity, and assume the v-groove opening angle is 90 deg, then the load between the sphere and the plane is 1155 N and the maximum contact stress is 1625MPa.

3.2 Sphere in cylinder

Consider a case where instead of v-grooves with flat faces, we imagine the v-grooves are cylinders with a radius close to that of the ball, but with the opposite sign. Assume steel materials and assume the ball radius is 12.5mm. Now with a load of 1000N, consider various cases for the cylinder radius

cylinder radius (mm)	Maximum Hertzian stress (MPa)
-12.6	764
-13	1054
-25	1981
-50	2243
-1000	2453
Flat plate	2464

For a sphere radius of 25mm we find

Cylinder radius (mm)	Maximum Hertzian stress (MPa)
-26	664
-30	927
-50	1248
Flat plate	1552

We have used the calculator http://www.mesys.ch/calc/hertz.fcgi.

Using cylinders as the faces of the v-grooves is quite advantageous, but only if the cylinder is close in radius to that of the sphere.

and d=7.9e-8m and a=3.15e-5m

For a cylinder on a plane,

$$F_c = \frac{\pi L R p_0^2}{E^*}$$

=547N

and d=1.27e-7m

a=4.0e-5m

Clearly the cylinder can take much higher loads. In practice for this to work we need good alignment between a cylinder and a V-groove. The contact width/L sets the alignment tolerances of the two parts.

4 Axial Support

We start with a formula from KOR 74

$$\delta_{rms} = \gamma_N \frac{q}{D}(\frac{A}{N})^2$$

$$q = \rho g h$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

and slopes scale as $N^{-3/2}$

for our problem

E=9.1e10 Pa

ν=0.24

h=0.05m

D=1.006e6Nm

A=0.431m^2

q=1.24e3 N/m^2

if N=3 then γ=5.7e-3 and

δ=145nm.

If N=6,

γ=2.5e-3 and

δ=15.9nm.

So in principle we only need 6 axial support points. This does not give the actual design.

FEA (shell model) give pv~74.3 nm and 75.3nm for the two clockings (2 supports on major dia, 2 on minor dia.)

The rms u slopes are 2.74e-7 rad and 2.56e-7 rad (total allowed 8.41e-7)

The rms v slopes are 2.48e-7 rad and 2.84e-7 rad (total allowed 1.19e-6)

This seems acceptable, particularly when we consider this is for full axial gravity.

If N=9

γ_9=3.1e-3,

N=12

γ_{12}=1.6e-3

δ=2.54nm.

from Cabak fea shell

pv~20.2nm

rms = 5.07 nm

rms u slope =6.97e-8 rad (total allowed 8.41e-7) rms v slope=1.04e-7 rad (total allowed 1.19e-6)

for N=6 expect slopes to be bigger by 2.83*2.5/1.6=4.42, so N=6 support uses almost all the total error budget for y slopes, so it probably wont work.

For brick model

Pv=27.2nm

Rms=6.13nm

Rms x slope=9.32e-8

Rms y slope=1.26e-7

(all this is a quarter model)

5 Lateral Support

We assume that the lateral support of the tertiary will be with a central diaphragm, similar to the lateral supports of the Keck segments and the Keck tertiary. This requires machining a blind hole in the center of the back of the mirror. This may cause local deformation from stress relief or polishing loads or gravity loads.

5.1 Center of gravity

The diaphragm should be located in the plane of the center of gravity. So we want to find the cg location. Define a coordinate system with z=0 in the mirror midplane. Call dz the distance of the cg on front of the midplane and u the depth of the hole past the midplane

$$[A_t h - A_l(h/2+u)]dz + A_l(h/2+u)(h/4+u/2-h/2) = 0$$

$$dz = \frac{A_l(h^2-4u^2)}{8[A_t h - A_l(h/2+u)]}$$

where $A_t = \pi ab$ = area of the tertiary

A_l = area of hole

Note this formula is correct for $u=\pm h/2$, and $A_t=A_l$.

For the full sized tertiary $A=1.255\text{m}^2$ the hole has a depth of 84.8mm into a mirror with a thickness of 125mm. The hole diameter is 254mm. The full tertiary mass is about 397 kg.

For the deployable tertiary $A_t=0.431\ \text{m}^2$, the thickness will be 50mm, with a hole depth and diameter to be determined. If we pick a=0.075m and u=11mm, $A_l=1.767\text{e-}2\ \text{m}^2$ and dz=0.21mm. The tertiary mass is about 55kg.

The glass mass of a segment is about 399 kg.

As the mirror at the center is significantly thinner its useful to see what the local gravity deformations are. For a clamped circular plate

$$z(r) = \frac{-q}{64D}(a^2-r^2)^2$$

$$s_x = \frac{qa^3}{16D}\frac{x}{a}(1-\rho^2)$$

so the maximum deflection is

$$z_{\max} = \frac{qa^4}{64D} = \frac{3\rho g(1-\nu^2)a^4}{16Eh^2}$$

and the maximum slope is

$$s_{\max} = \frac{qa^3}{24\sqrt{3}D} = \frac{\rho g(1-\nu^2)a^3}{2\sqrt{3}Eh^2}$$

Since the slopes are antisymmetric, the peak to peak is 2x this.

now

q=ρgh

D=Eh^3/12(1-ν^2)

Average deflection $= z_{avg} = \frac{1}{3}\frac{qa^4}{64D}$

The rms deflection is $\sigma_{defl} = \sqrt{\frac{4}{45}}\frac{qa^4}{64D}$

The rms x slope is $\sigma_{xslope} = \frac{qa^3}{32\sqrt{3}D}$

For h= 0.01m

a=0.1m

dz=0.53mm

q=2.48e2

D=8.05e3

z_{max}=48 nm

s_{max}=7.41e-7 rad

$\sigma_{xslope=}$=5.56e-7 rad

for h=0.014m

a=0.075m

dz= 0.21 mm

z_{max}=7.7 nm

s_{max}=1.59e-7

$\sigma_{xslope=}$=1.20e-7 rad

E=9.1e10 Pa

ρ=2.53e3kg/m3

ν=0.24

h=0.05m

D=1.006e6Nm

A=0.431m^2

q=1.24e3 N/m^2

suppose we want to support the mirror kinematically, and we want these 3 points to be separated by 120 deg , and be on a circle centered on the tertiary (this causes the axial forces to be equal)

If one of the supports lies on the symmetry axis and serves as the pivot point for deployment/retraction then the circle has a radius of d/sqrt(2) = 1100/sqrt(2)=777 mm